Analyzing the Resting Brain With Dynamic Mode Decomposition

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Advances in Dynamic Graphs:
Algorithms, Applications, and Challenges
SIAM CSE 2017

Overview

1. Motivation and Data

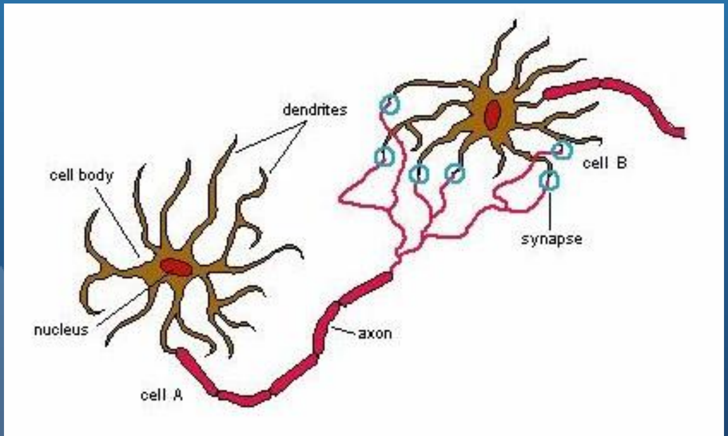
1. Overview of Dynamic Mode Decomposition

1. Preliminary Results

1. Goals

Motivation and Data

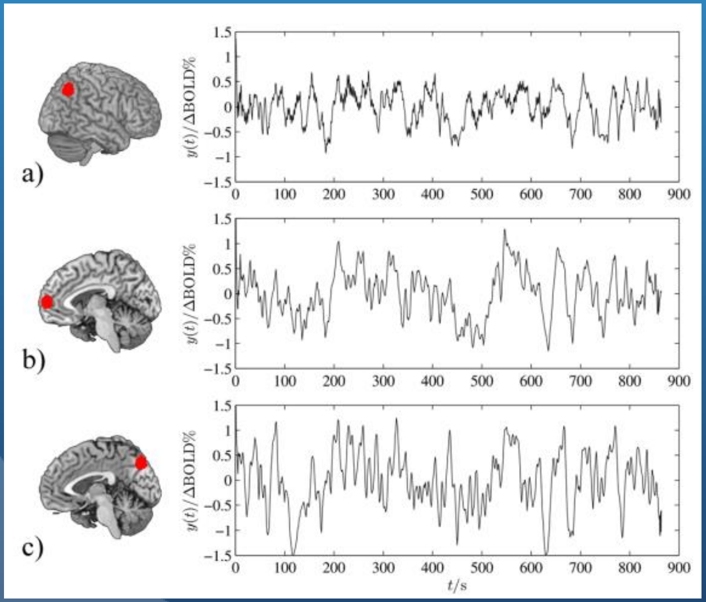
- Average adult human brain has roughly 100 billion neurons
- Communicate via electro-chemical signaling at synapses
- Synapses exist in grey matter at cortex (surface) and subcortical nuclei
- We can measure proxies of neuronal activity via MRI (and others: EEG, ECog, MEG)



1. Hseih-Wilson Group, Cal-Tech. http://chemistry.caltech.edu/fucose/

Motivation and Data

- Resting-state fMRI
  - Blood-oxygenation level dependent (BOLD) signal at cortex at rest
  - BOLD signal correlates with neuronal activity, measured over time



1. Minati, L., et al., *Synchronization, non-linear dynamics and low-frequency fluctuations: analogy between spontaneous brain activity and networked single-transistor chaotic oscillators*. Chaos, 2015. **25**(3): p. 033107.

Motivation and Data

- Human Connectome Project
  - Large scale study of the human brain
  - Washington University in St. Louis and University of Minnesota
  - Very high quality, with high resolution in space and time
  - For our purposes, resting state data has:
    - ~15 minute acquisitions
      - 1200 time points, 0.72 Hz
    - ~32k surface points per per time slice

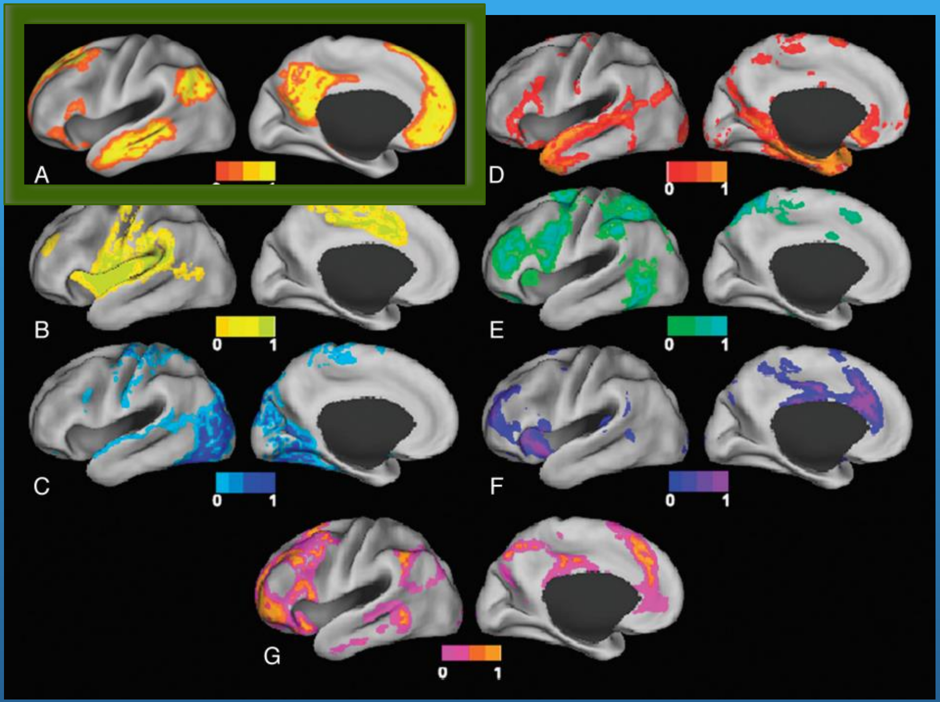
1. Glasser, M. et al. The Minimal PreProcessing Pipeline For The Human Connectome Project. Neuroimaging, 2013 Oct 15.

Motivation and Data

- Resting-state fMRI
  - PCA and ICA
    - Identify spatial modes with temporally correlated activity
  - Seed-based correlations
  - Fourier analyses

Resting State with DMD

- Decompose our signal into resting state networks
    - ~10 networks known, anatomy and function
    - 0.01 Hz – 0.1 Hz
    - Default Mode Network (DMN) most consistent
- Can DMD find these networks?



a. Default mode network b. somato-motor network c. visual network d. language network e. dorsal attention network f. ventral attention network g. frontoparietal control network

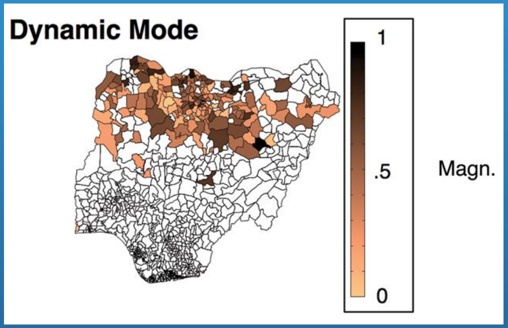
1. Lee, M. et al. Resting-State fMRI: A Review of Methods and Clinical Applications. ADJNR, 2013

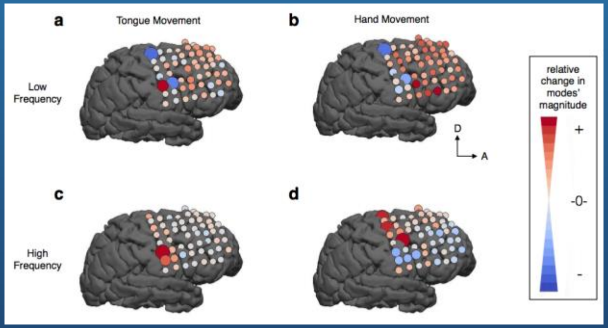
Dynamic Mode Decomposition

- Modal decomposition algorithm developed for the analysis of dynamic fluid flows

- Able to extract both spatial and temporal information

- Applied to a variety of fields:
  - Video processing
  - Control systems
  - Disease mapping
  - EEG sleep spindle analysis
  - Fincancial trading





1. Schmid, P. Dynamic Mode Decomposition of Numerical And Experimental Data. Journal of Fluid Mechanics, 2010.
2. Proctor, J. et al. Discovery Dynamic patterns from infectious disease data using dynamic mode decomposition. Int. Health, 2015.
3. Brunton, B. et al. Extracting spatio-temporal coherent patterns in large-scale neural recordings using DMD. arXiv, 2014.

Dynamic Mode Decomposition

Assume we have some evolving system with state X, and don't know the governing equation f, and our update matrix A is unknown

$$\frac{d\overrightarrow{x}}{dt} = f(\overrightarrow{x}, t) \approx A\overrightarrow{x}$$

Can sample system to generate a matrix of measurements

$$X = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \dots & x_m \\ | & | & & | \end{bmatrix}$$

1. Schmid, P. Dynamic Mode Decomposition of Numerical And Experimental Data. Journal of Fluid Mechanics, 2010.

Dynamic Mode Decomposition

We can split our sample matrix into two m-1 dimensional matrices

$$X_1 = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \dots & x_{m-1} \\ | & | & & | \end{bmatrix} \quad X_2 = \begin{bmatrix} | & | & & | \\ x_2 & x_3 & \dots & x_m \\ | & | & & | \end{bmatrix}$$

and can then construct A then so that

$$X_2 = AX_1 \rightarrow A = X_2 X_1^{\dagger}$$

DMD tries to fit a curve to the best **linear** dynamical system that accounts for the evolution of our system (even if the system is non-linear)

1. Schmid, P. Dynamic Mode Decomposition of Numerical And Experimental Data. Journal of Fluid Mechanics, 2010.

Dynamic Mode Decomposition: Algorithm

If our system is defined as

$$X_2 = AX_1$$

we can can compute the SVD of X_1 and rewrite it as

$$X_2 = AU\Sigma V^*$$

and then solve for A via

$$A = X_2 V \Sigma^{-1} U^*$$

giving us the pseudoinverse formulation

1. Schmid, P. Dynamic Mode Decomposition of Numerical And Experimental Data. Journal of Fluid Mechanics, 2010.

Dynamic Mode Decomposition: Algorithm

Can transform A via similarity transform:

$$\tilde{A} = U^* A U = U^* X_2 V \Sigma^{-1}$$

Since we're interested in system evolution → compute the eigendecomposition of Ã

$$\tilde{A} W = W \Lambda$$

where eigenvalues of **Ã** are eigenvalues of A, and DMD modes are:

$$\Phi = UW \quad \text{or} \quad \boxed{\Phi = X_2 V \Sigma^{-1} W}$$

1. Schmid, P. Dynamic Mode Decomposition of Numerical And Experimental Data. Journal of Fluid Mechanics, 2010.

Dynamic Mode Decomposition: Algorithm

Began with goal of modeling how system evolves using a first-order model

$$\frac{d\vec{x}}{dt} = f(\vec{x}, t) \approx A\vec{x}$$

Can solve for state of system at any time with well-known solution:

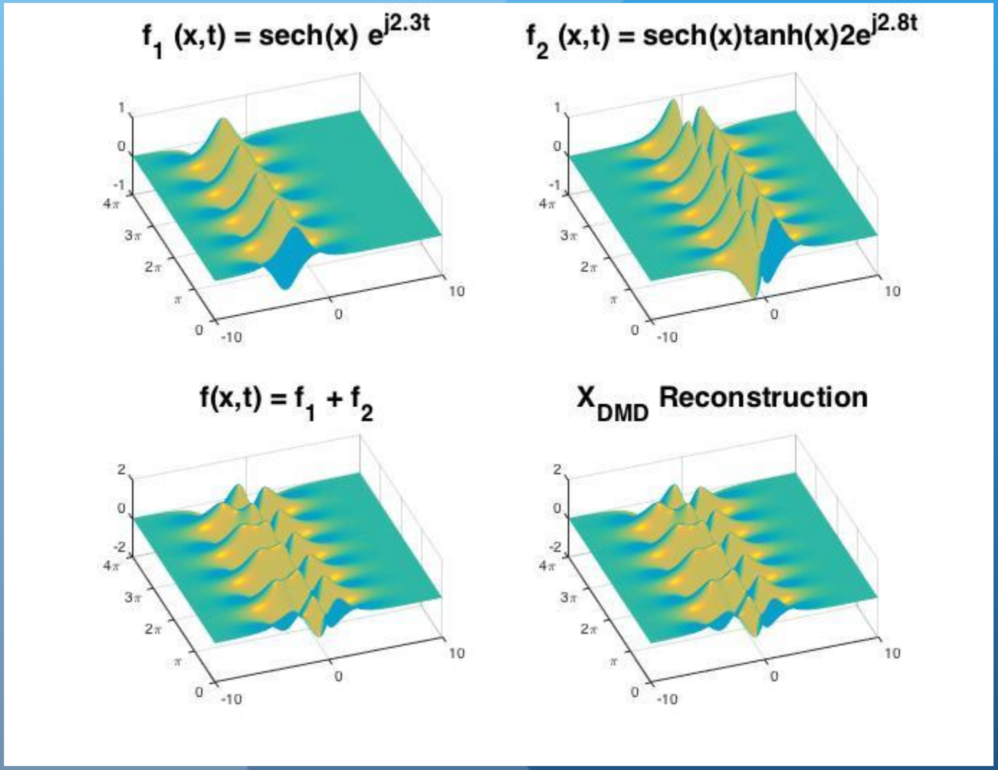
$$x(t) = \sum_{k=1}^{r} \phi_k exp(\omega_k t) b_k = \Phi exp(\Omega t) b$$

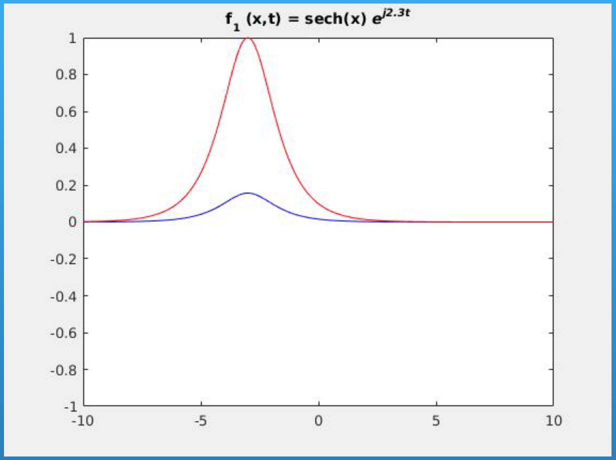
where Φ_k are DMD modes, ω_k are transformed eigenvalues, and b is vector of mode amplitudes

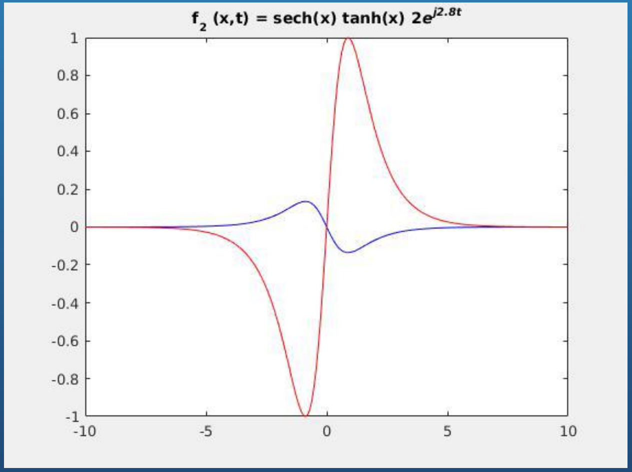
$$\omega_k = \frac{ln(\lambda_k)}{\Delta t}$$

1. Schmid, P. Dynamic Mode Decomposition of Numerical And Experimental Data. Journal of Fluid Mechanics, 2010.

Dynamic Mode Decomposition: Example



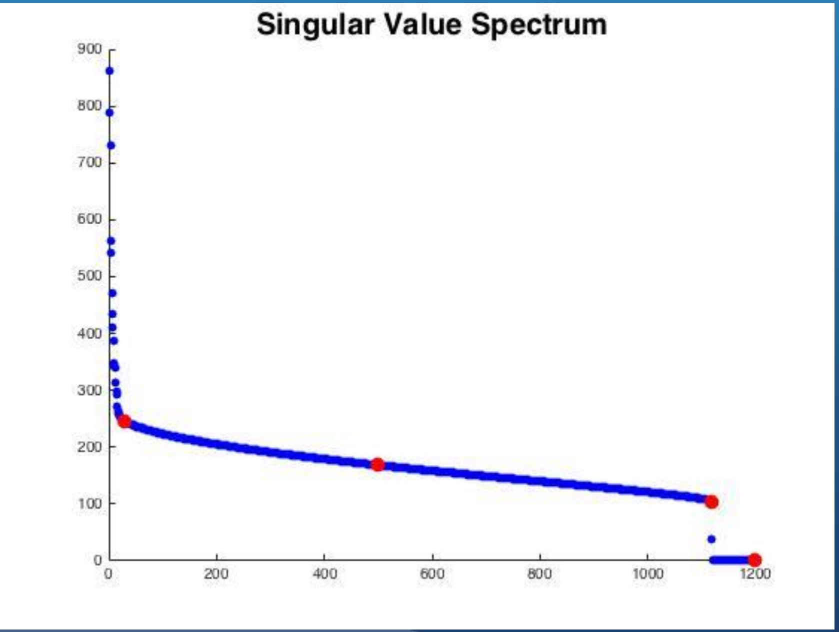




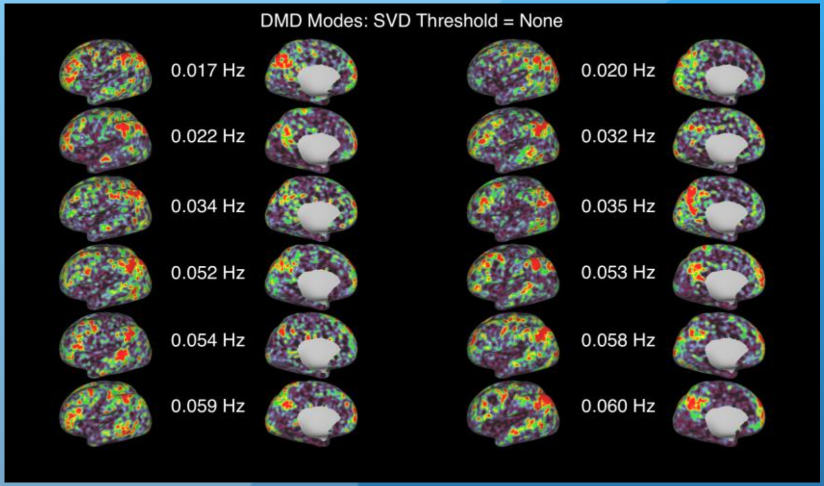
1. Kutz, N., Brunton, S., Brunton, B., Proctor J., *Dynamic Mode Decomposition: Data-Driven Modeling of Complex Systems.*

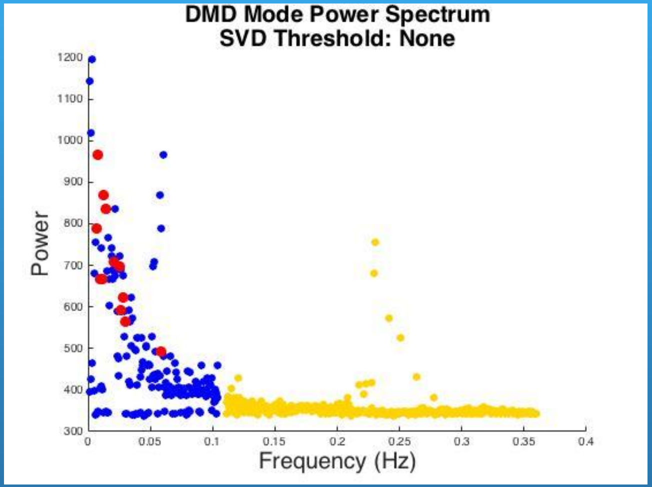
Resting State with DMD

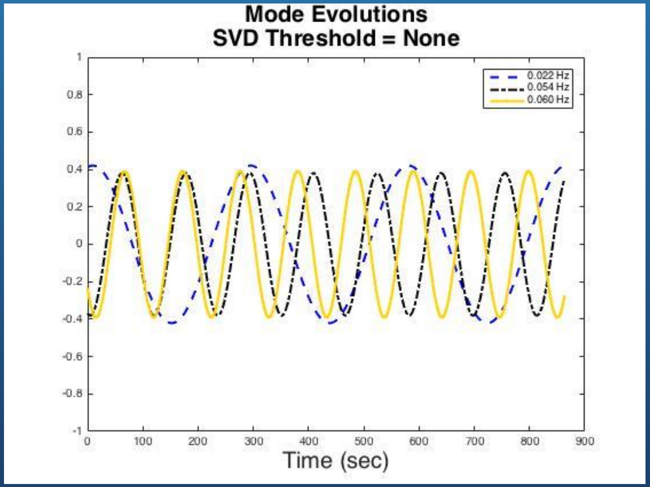
- SVD shows possible low-rank structure, motivates rank-truncation tests
- Test ranks of 500, 1118, and full spectrum
  - “elbow” ~ rank 30 showed no coherent DMD modes
- Examine power spectrum, DMD modes, and mode dynamics



Resting State with DMD: Full Rank



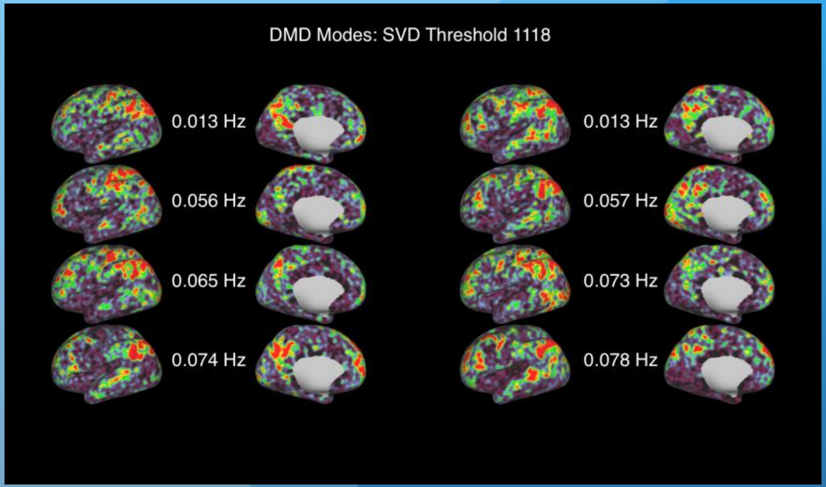


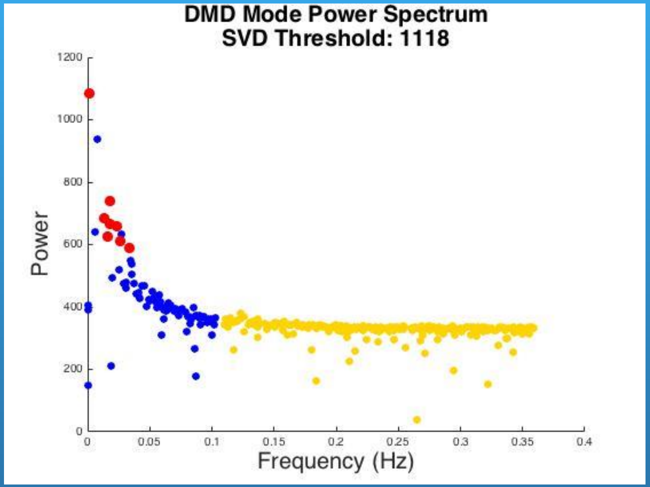


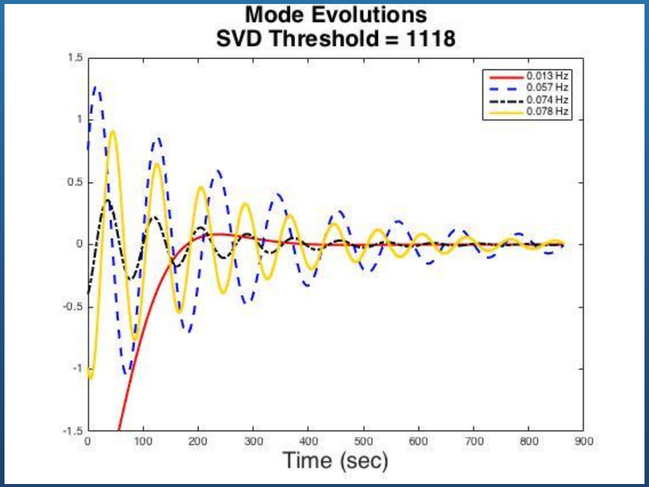
- Stable over time
- High power, low frequency

- Follow spatial distribution of Default Mode Network

Resting State with DMD: Rank = 1118



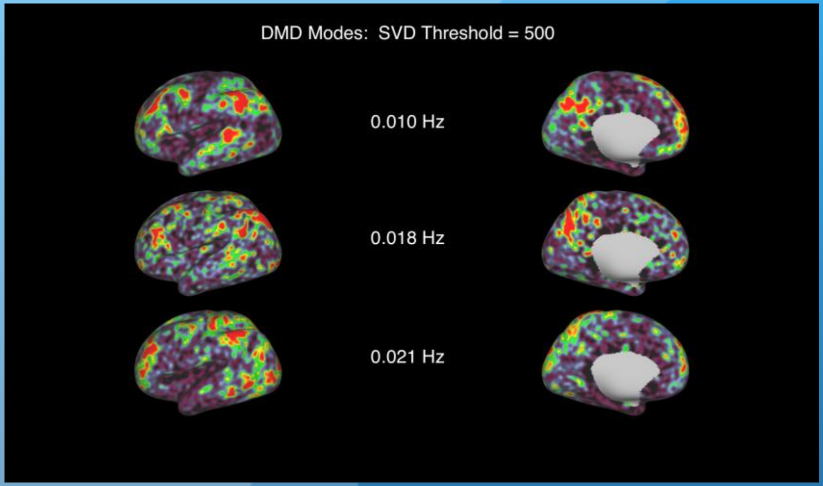


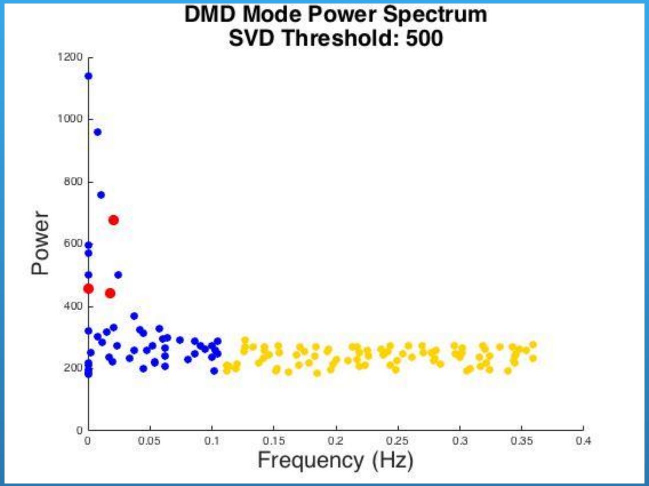


- Gradually decay
- Few of anatomical relevance
- High power, low frequencies

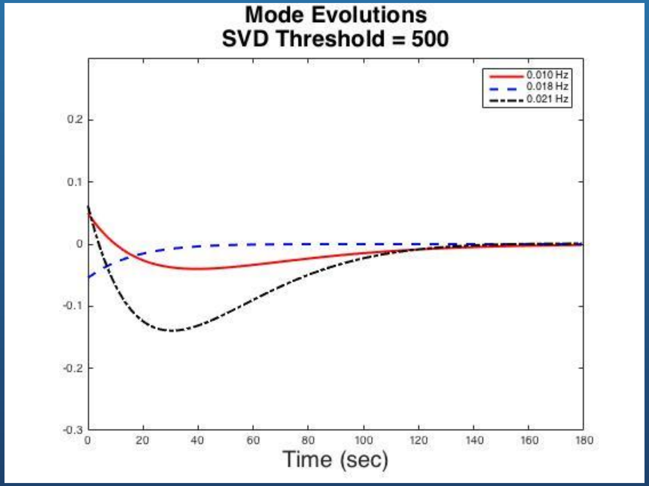
- Default Mode Network
- Hints of Dorsal Attention Network

Resting State with DMD: Rank = 500





- All modes decay quickly
- Fewer modes look anatomically relevant
  - Too much truncation?
- Primarily Default mode Network



Resting State with DMD: Trends

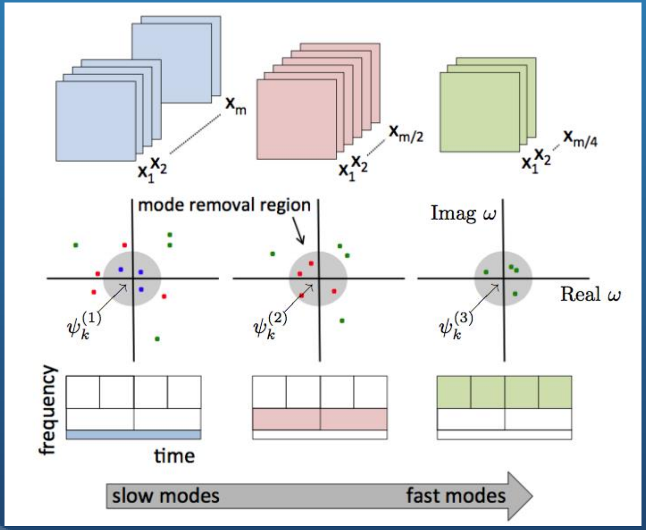
- Default Mode Network seems independent of rank, little evidence of other networks
- Coherent DMD modes tend to have high power, lower frequencies
  - → automate detection
- SVD shows low-rank spatial structure
  - Assumed stationarity → "average" spatial maps
  - Transient signals that arent oscillatory might manifest as noise
- Perhaps no low rank structure in this basis
  - Increased rank truncation produces more-quickly decaying modes
    - Poor signal reconstruction
    - How to interpret this biologically?

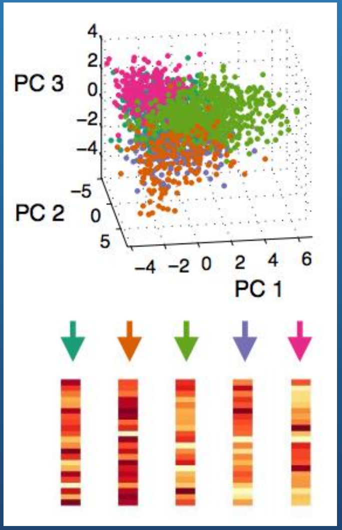
Resting State with DMD: Goals

- Sliding window DMD
  - Build DMD mode libraries, clustering / classification

- Multi-Resolution DMD
  - Recursivley remove low-frequency modes

- Population-based analysis on DMD modes
  - Hierarchical models of spatial modes to account for subject variability

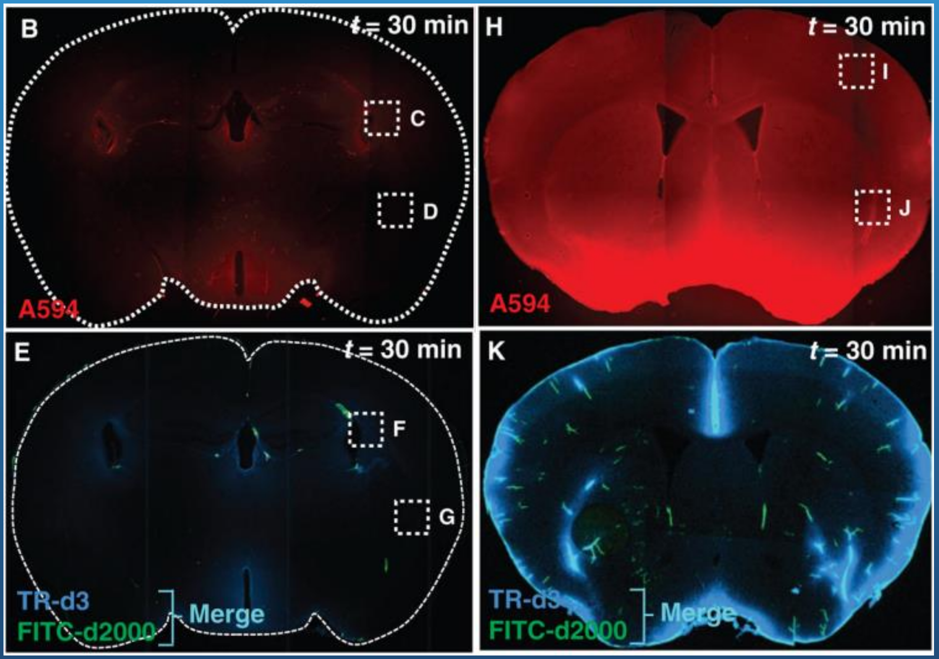




1. Kutz, J. et al. Multi-Resolution Dynamic Mode Decomposition. SIAM J. Applied Dynamical Systems. 2016.
2. Brunton, B. Extracting spatial-temporal coherent patterns in large-scale neural recordings using DMD. arXiv, 2014.

Resting State with DMD: Goals

- Analyze brain “glymphatic” (lymphatic) system
- Very slow oscillatory pattern
- Hypothesis:
    - Disrupted in Alzheimer’s disease
- Driven by simple diffusion and vasculature pulsations in sulci (brain folds)
- Are oscillations even detectable with imaging?



1. Iliff, J. et al. A Paravascular Pathway Fascillitates CSF Flower Through The Brain Paranechyma and the Clearance of Interstitial Solutes, Including Amyloid Beta. Sci. Transl. Med. 2012.

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Appendix: Discrete Eigenvalue Plots