Tensor Decompositions for Analyzing Multi-link Graphs

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Sandia National Laboratories

SIAM Parallel Processing for Scientific Computing
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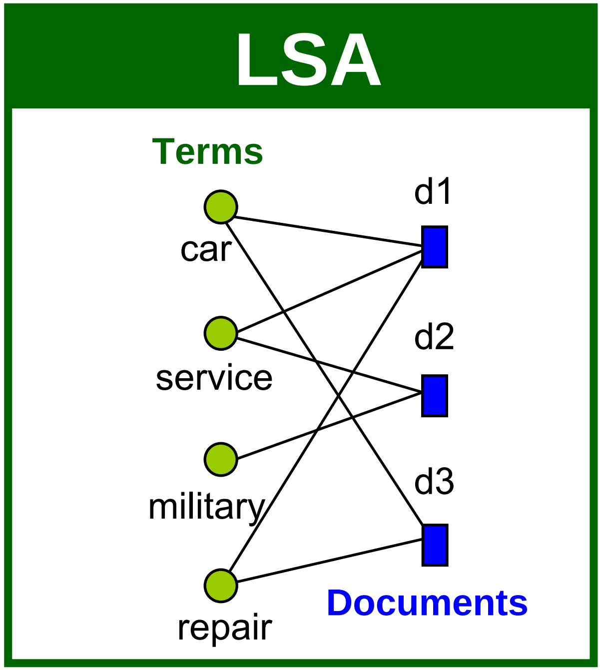
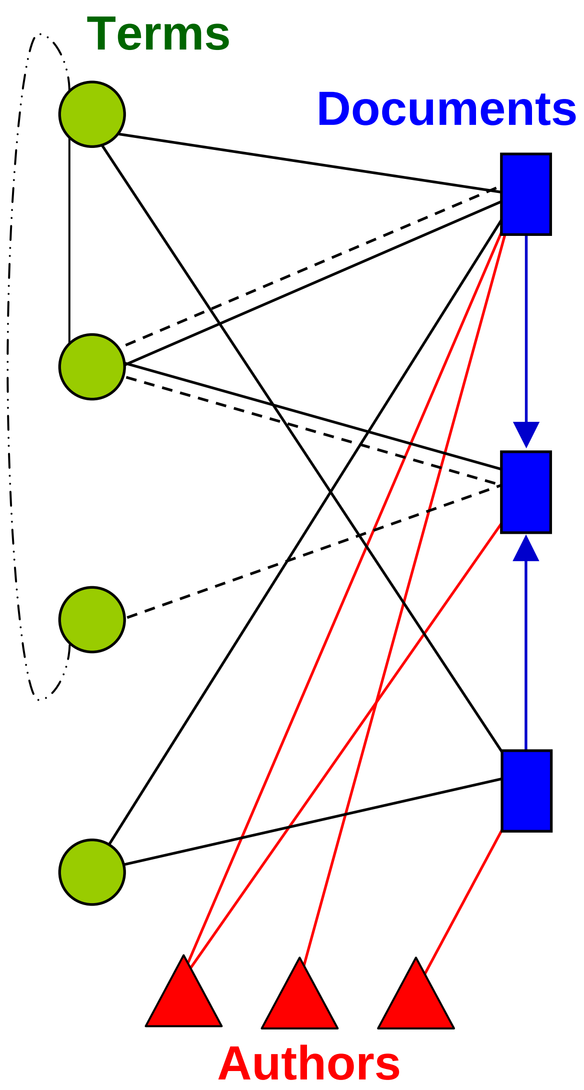
Linear Algebra, Graph Analysis, Informatics

- **Network analysis and bibiliometrics**
- **PageRank** [Brin & Page, 1998]
  - Transition matrix of a Markov chain: $\mathbf{M}$
  - Ranks: $\mathbf{M}\mathbf{r} = \mathbf{r}$
- **HITS** [Kleinberg, 1998]
  - Adjacency matrix of the Web graph: $\mathbf{L}$
  - Hubs: $\mathbf{L}\mathbf{L}^T\mathbf{y} = \mathbf{y}$
  - Authorities: $\mathbf{L}^T\mathbf{L}\mathbf{x} = \mathbf{x}$



- **Latent Semantic Analysis (LSA)** [Dumais, et al., 1988]
  - Vector space model of documents (term-document matrix): $\mathbf{A}$
  - Truncated SVD: $\mathbf{A} \approx \mathbf{T}\boldsymbol{\Sigma}\mathbf{D}^{\top} = \sum_{r=1}^{k} \sigma_r \, \mathbf{t}_r \mathbf{d}_r^T$
  - Maps terms and documents to the “same” *k*-dimensional space

Multi-Link Graphs



- **Nodes (one type) connected by multiple types of links**
  - Node x Node x Connection
- **Two types of nodes connected by multiple types of links**
  - Node A x Node B x Connection
- **Multiple types of nodes connected by multiple types of links**
  - Node A x Node B x Node C x Connection
  - Directed and undirected links

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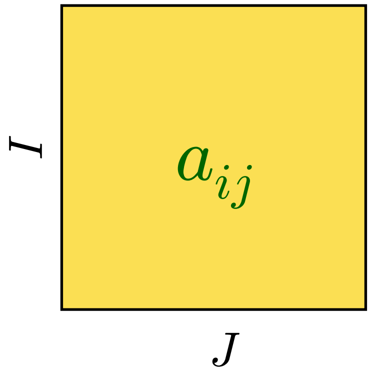
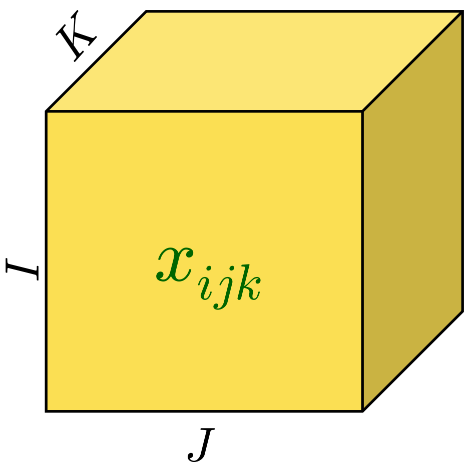
Tensors

Notation	
s	scalar
$\mathbf{a}$	vector
$\mathbf{B}$	matrix
$\mathcal{X}$	tensor

An I x J matrix



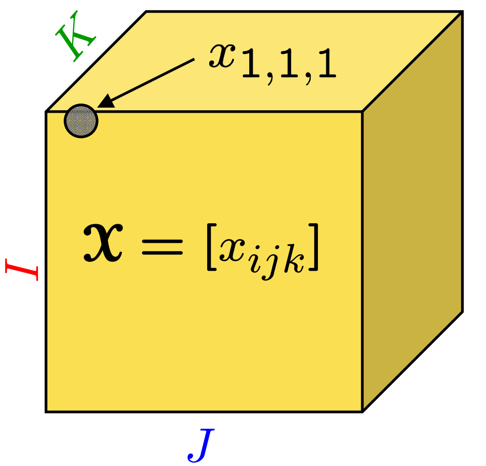
An $I \times J \times K$ tensor



- **Other names for tensors**
  - Multidimensional array
  - N-way array
- **Tensor order**
  - Number of dimensions
- **Other names for dimension**
  - Mode
  - Way
- **Example**
  - The matrix **A** (at left) has order 2.
  - The tensor $\mathcal{X}$ (at left) has order 3 and its 3rd mode is of size K.

Tensors

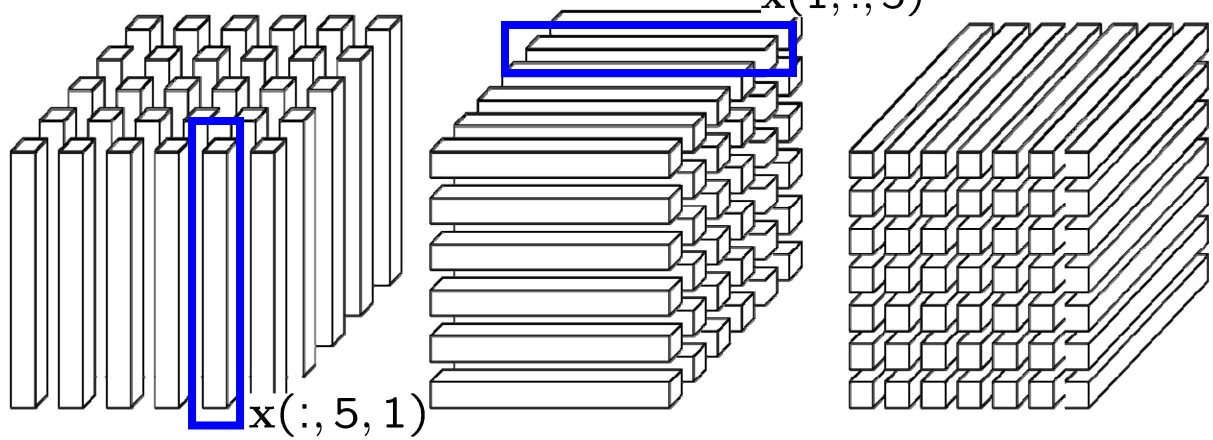
An $I \times J \times K$ tensor



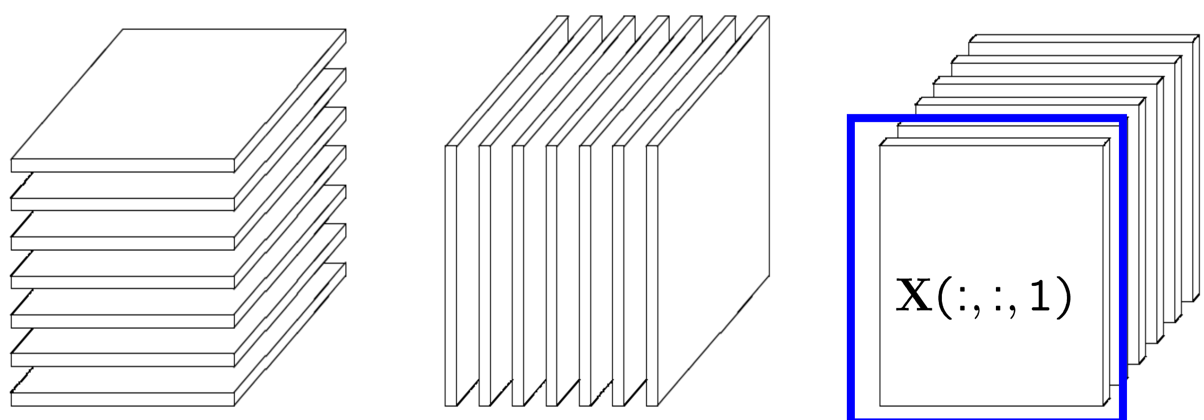
3[rd] order tensor

mode 1 has dimension I
mode 2 has dimension J
mode 3 has dimension K

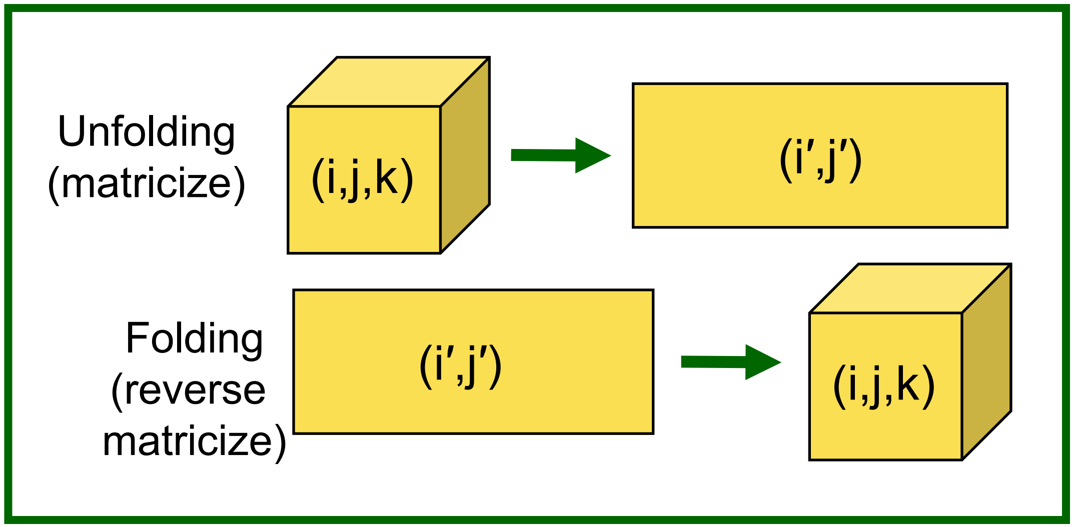
Column (Mode-1) Fibers | Row (Mode-2) Fibers | Tube (Mode-3) Fibers



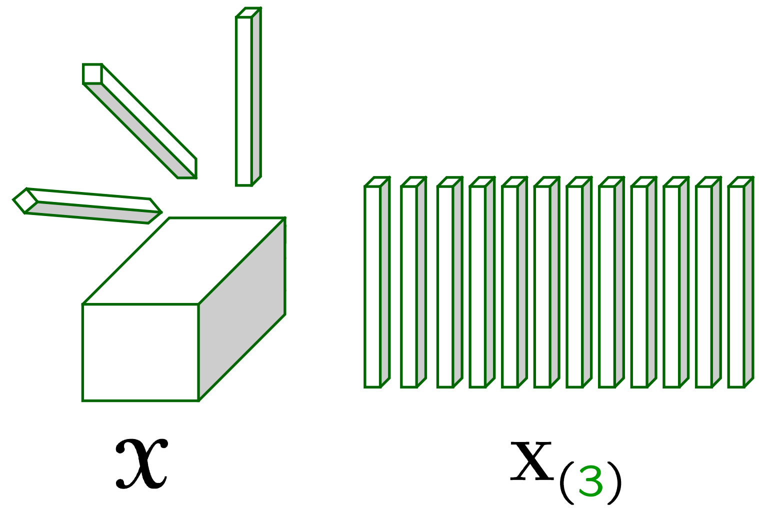
Horizontal Slices | Lateral Slices | Frontal Slices



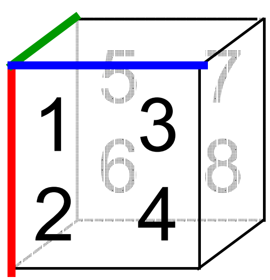
Tensor Unfolding:
Converting a Tensor to a Matrix



$\mathbf{X}_{(n)}$: The mode-**n** fibers are rearranged to be the columns of a matrix



$\mathcal{X} =$



$$\mathbf{X}_{({\color{red}1})} = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix}$$

$$\mathbf{X}_{({\color{blue}2})} = \begin{bmatrix} 1 & 2 & 5 & 6 \\ 3 & 4 & 7 & 8 \end{bmatrix}$$

$$\mathbf{X}_{({\color{green}3})} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

$$\mathrm{vec}(\mathcal{X}) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}^{\mathsf{T}}$$

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Tensor n-Mode Multiplication

$$\mathcal{X} \in \mathbb{R}^{I \times J \times K}, \ \mathbf{B} \in \mathbb{R}^{M \times J}, \ \mathbf{a} \in \mathbb{R}^{I}$$

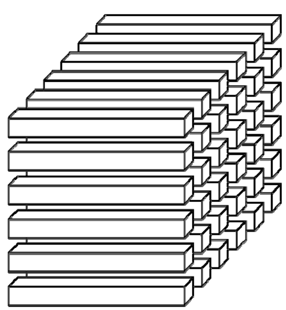
Tensor Times Matrix

$$\mathcal{Y} = \mathcal{X} \times_2 \mathrm{B} \in \mathbb{R}^{I \times M \times K}$$

$$y_{imk} = \sum_j x_{ijk} \, b_{mj}$$

$$\mathbf{Y}_{(2)} = \mathbf{B}\mathbf{X}_{(2)}$$

Multiply each row (mode-2) fiber by **B**

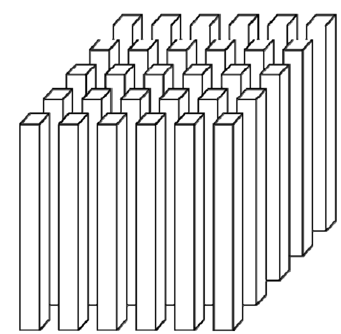


Tensor Times Vector

$$\mathbf{Y} = \mathcal{X} \,\bar{\times}_1\, \mathbf{a} \in \mathbb{R}^{J \times K}$$

$$y_{jk} = \sum_i x_{ijk} \, a_i$$

Compute the dot product of **a** and each column (mode-1) fiber

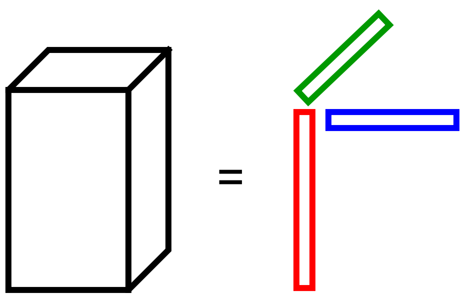


Outer, Kronecker, & Khatri-Rao Products

3-Way Outer Product

$$\mathcal{X} = \mathbf{a} \circ \mathbf{b} \circ \mathbf{c}$$

$$x_{ijk} = a_i b_j c_k$$



Rank-1 Tensor

Matrix Kronecker Product

$$\underset{M \times N}{\mathbf{A}} \otimes \underset{P \times Q}{\mathbf{B}} = \underbrace{\begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \cdots & a_{1N}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \cdots & a_{2N}\mathbf{B} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1}\mathbf{B} & a_{M2}\mathbf{B} & \cdots & a_{MN}\mathbf{B} \end{bmatrix}}_{MP \times NQ}$$

$$= \begin{bmatrix} \mathbf{a}_1 \otimes \mathbf{b}_1 & \mathbf{a}_1 \otimes \mathbf{b}_2 & \cdots & \mathbf{a}_N \otimes \mathbf{b}_Q \end{bmatrix}$$

Matrix Khatri-Rao Product

$$\underset{M \times R}{\mathbf{A}} \odot \underset{N \times R}{\mathbf{B}} = \underbrace{\begin{bmatrix} \mathbf{a}_1 \otimes \mathbf{b}_1 & \mathbf{a}_2 \otimes \mathbf{b}_2 & \cdots & \mathbf{a}_R \otimes \mathbf{b}_R \end{bmatrix}}_{MN \times R}$$

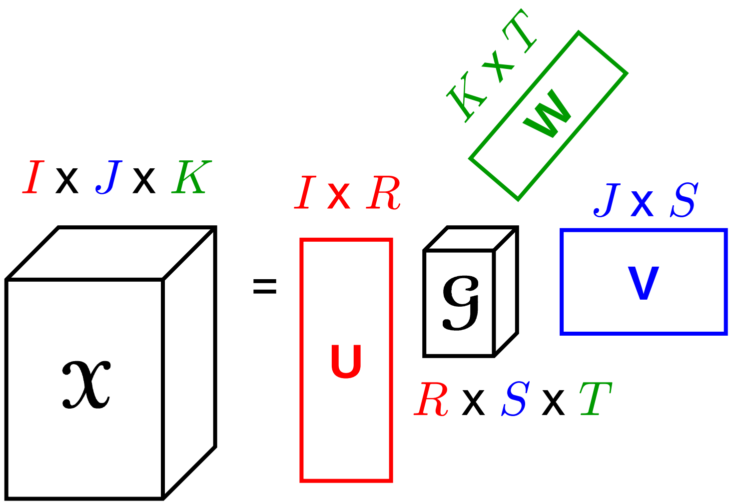
<u>Observe</u>: For two vectors **a** and **b**, **a** $\circ$ **b** and **a** $\otimes$ **b** have the same elements, but one is shaped into a matrix and the other into a vector.

Specially Structured Tensors

Tucker Tensor

$$
\begin{aligned}
\mathcal{X} &= \mathcal{G} \times_1 \mathbf{U} \times_2 \mathbf{V} \times_3 \mathbf{W} \\
&= \sum_r \sum_s \sum_t g_{rst} \; \mathbf{u}_r \circ \mathbf{v}_s \circ \mathbf{w}_t \\
&\equiv [\![\mathcal{G} \, ; \mathbf{U}, \mathbf{V}, \mathbf{W}]\!]
\end{aligned}
$$

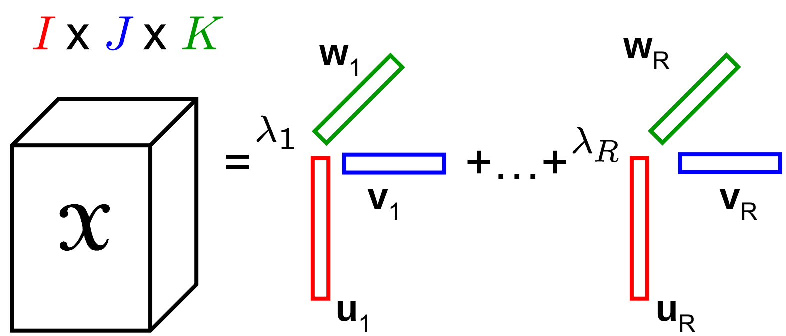
Our Notation



Kruskal Tensor

$$
\begin{aligned}
\mathcal{X} &= \sum_r \lambda_r \; \mathbf{u}_r \circ \mathbf{v}_r \circ \mathbf{w}_r \\
&\equiv [\![\boldsymbol{\lambda} \, ; \mathbf{U}, \mathbf{V}, \mathbf{W}]\!]
\end{aligned}
$$

Our Notation



Specially Structured Tensors

Tucker Tensor

$$\mathcal{X} = \mathcal{G} \times_1 \mathbf{U} \times_2 \mathbf{V} \times_3 \mathbf{W}$$
$$= \sum_r \sum_s \sum_t g_{rst}\, \mathbf{u}_r \circ \mathbf{v}_s \circ \mathbf{w}_t$$
$$\equiv [\![\mathcal{G} \,; \mathbf{U}, \mathbf{V}, \mathbf{W}]\!]$$

In matrix form:

$$\mathbf{X}_{(1)} = \mathbf{U}\mathbf{G}_{(1)}(\mathbf{W} \otimes \mathbf{V})^{\mathsf{T}}$$
$$\mathbf{X}_{(2)} = \mathbf{V}\mathbf{G}_{(2)}(\mathbf{W} \otimes \mathbf{U})^{\mathsf{T}}$$
$$\mathbf{X}_{(3)} = \mathbf{W}\mathbf{G}_{(3)}(\mathbf{V} \otimes \mathbf{U})^{\mathsf{T}}$$

$$\mathrm{vec}(\mathcal{X}) = (\mathbf{W} \otimes \mathbf{V} \otimes \mathbf{U})\mathrm{vec}(\mathcal{G})$$

Kruskal Tensor

$$\mathcal{X} = \sum_r \lambda_r\, \mathbf{u}_r \circ \mathbf{v}_r \circ \mathbf{w}_r$$
$$\equiv [\![\boldsymbol{\lambda} \,; \mathbf{U}, \mathbf{V}, \mathbf{W}]\!]$$

In matrix form:

Let $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda)$

$$\mathbf{X}_{(1)} = \mathbf{U}\,\boldsymbol{\Lambda}\,(\mathbf{W} \odot \mathbf{V})^{\mathsf{T}}$$
$$\mathbf{X}_{(2)} = \mathbf{V}\,\boldsymbol{\Lambda}\,(\mathbf{W} \odot \mathbf{U})^{\mathsf{T}}$$
$$\mathbf{X}_{(3)} = \mathbf{W}\,\boldsymbol{\Lambda}\,(\mathbf{V} \odot \mathbf{U})^{\mathsf{T}}$$

$$\mathrm{vec}(\mathcal{X}) = (\mathbf{W} \odot \mathbf{V} \odot \mathbf{U})\,\lambda$$

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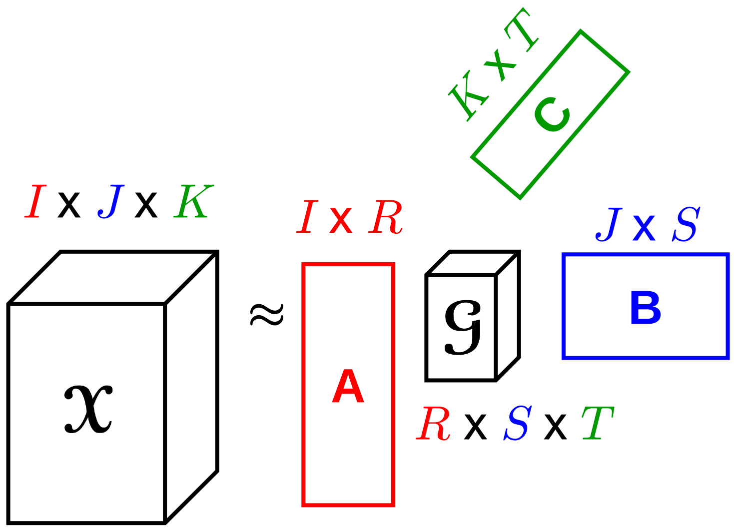
Tucker Decomposition

Three-mode factor analysis	[Tucker, 1966]
Three-mode PCA	[Kroonenberg, et al. 1980]
***N*-mode PCA**	[Kapteyn et al., 1986]
Higher-order SVD (HO-SVD)	[De Lathauwer et al., 2000]



$$\mathcal{X} \approx [\![\mathcal{G} \,;\, \mathbf{A}, \mathbf{B}, \mathbf{C}]\!]$$

Given **A**, **B**, **C**, the optimal core is:

$$\mathcal{G} = [\![\mathcal{X} \,;\, \mathbf{A}^\dagger, \mathbf{B}^\dagger, \mathbf{C}^\dagger]\!]$$

- **A**, **B**, and **C** may be orthonormal (generally, full column rank)
- $\mathcal{G}$ is <u>not</u> diagonal
- Decomposition is **not** unique

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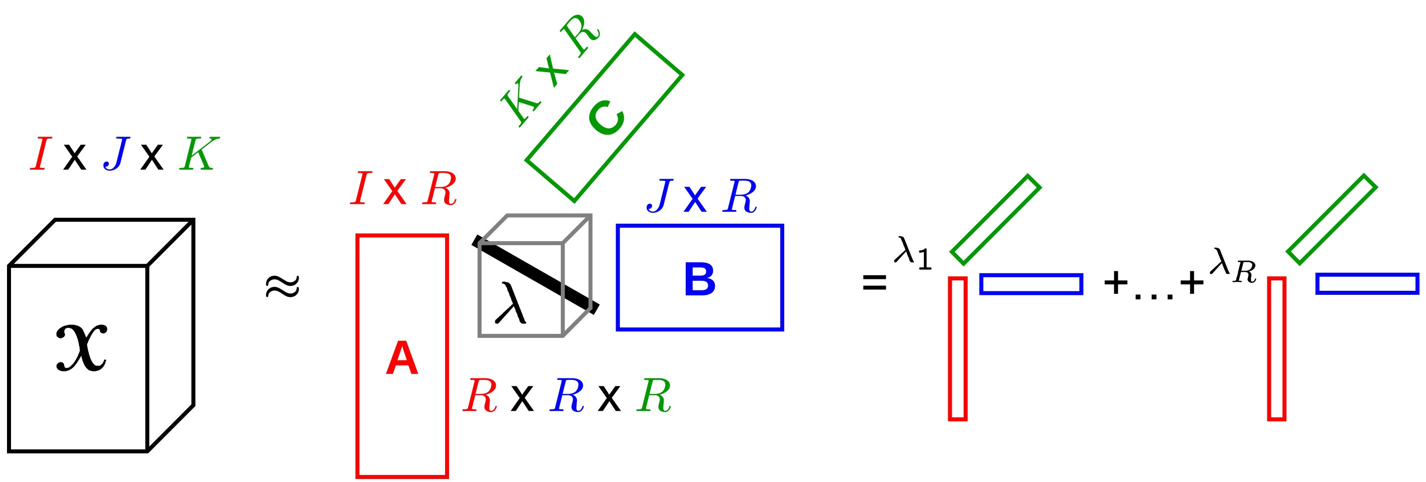
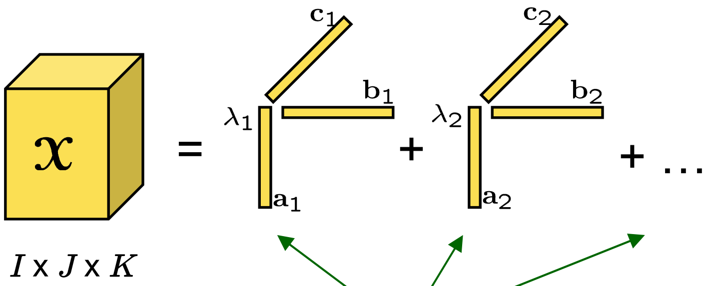
CANDECOMP/PARAFAC (CP) Decomposition



$$\mathcal{X} \approx [\![\boldsymbol{\lambda} \,;\, \mathbf{A}, \mathbf{B}, \mathbf{C}]\!] = \sum_r \lambda_r \, \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$$

- **CANDECOMP** = Canonical Decomposition [Carroll & Chang, 1970]
- **PARAFAC** = Parallel Factors [Harshman, 1970]
- Core is *diagonal* (specified by the vector λ)
- Columns of **A**, **B**, and **C** are **not** orthonormal

CP Alternating Least Squares (CP-ALS)



Find all the vectors in one direction simultaneously

Khatri-Rao Pseudoinverse

$$(\mathbf{B} \odot \mathbf{A})^\dagger = \left(\left(\mathbf{B}^\mathsf{T}\mathbf{B} \right) * \left(\mathbf{A}^\mathsf{T}\mathbf{A} \right) \right)^\dagger (\mathbf{B} \odot \mathbf{A})^\mathsf{T}$$

- Fix **A**, **B**, and Λ, solve for **C**:

$$\min_{\mathbf{C}} \| \mathcal{X} - [\![\boldsymbol{\lambda} \, ; \mathbf{A}, \mathbf{B}, \mathbf{C}]\!] \| \equiv \min_{\mathbf{C}} \left\| \mathbf{X}_{(3)} - \mathbf{C}\boldsymbol{\Lambda} \, (\mathbf{B} \odot \mathbf{A})^\mathsf{T} \right\|$$

[Smilde et al., 2004]

- Optimal **C** is the least squares solution:

$$\mathbf{C} \; = \; \mathbf{X}_{(3)} \, (\mathbf{B} \odot \mathbf{A}) \left(\left(\mathbf{B}^\mathsf{T}\mathbf{B} \right) * \left(\mathbf{A}^\mathsf{T}\mathbf{A} \right) \right)^\dagger \boldsymbol{\Lambda}^{-1}$$

- Successively solve for each component (**C,B,A**)

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CP Alternating Least Squares (CP-ALS)

CP-ALS $(\mathcal{X}, R, M, \epsilon)$

$m \leftarrow 0$
$\mathbf{A} \leftarrow R$ principal eigenvectors of $\mathbf{X}_{(1)}\mathbf{X}_{(1)}^{\mathsf{T}}$
$\mathbf{B} \leftarrow R$ principal eigenvectors of $\mathbf{X}_{(2)}\mathbf{X}_{(2)}^{\mathsf{T}}$
repeat
    $m \leftarrow m + 1$
    $\mathbf{C} \leftarrow \mathbf{X}_{(3)}(\mathbf{B} \odot \mathbf{A})\left(\left(\mathbf{B}^{\mathsf{T}}\mathbf{B}\right) * \left(\mathbf{A}^{\mathsf{T}}\mathbf{A}\right)\right)^{\dagger}$
    Normalize columns of $\mathbf{C}$ to length 1
    $\mathbf{B} \leftarrow \mathbf{X}_{(2)}(\mathbf{C} \odot \mathbf{A})\left(\left(\mathbf{C}^{\mathsf{T}}\mathbf{C}\right) * \left(\mathbf{A}^{\mathsf{T}}\mathbf{A}\right)\right)^{\dagger}$
    Normalize columns of $\mathbf{B}$ to length 1
    $\mathbf{A} \leftarrow \mathbf{X}_{(1)}(\mathbf{C} \odot \mathbf{B})\left(\left(\mathbf{C}^{\mathsf{T}}\mathbf{C}\right) * \left(\mathbf{B}^{\mathsf{T}}\mathbf{B}\right)\right)^{\dagger}$
    Store column norms of $\mathbf{A}$ in $\boldsymbol{\lambda}$ and
    normalize columns of $\mathbf{A}$ to length 1
until $m > M$ or $\| \mathcal{X} - [\![\boldsymbol{\lambda} \,;\, \mathbf{A}, \mathbf{B}, \mathbf{C}]\!] \| < \epsilon$
return $\boldsymbol{\lambda} \in \mathbb{R}^{R}$; $\mathbf{A} \in \mathbb{R}^{I \times R}$; $\mathbf{B} \in \mathbb{R}^{J \times R}$; $\mathbf{C} \in \mathbb{R}^{K \times R}$
    such that $\mathcal{X} \approx [\![\boldsymbol{\lambda} \,;\, \mathbf{A}, \mathbf{B}, \mathbf{C}]\!]$

Analyzing SIAM Publication Data

1999-2004
SIAM Journal Data
5022 Documents

Tensor Toolbox (MATLAB)
[T. Kolda, B. Bader, 2007]

Google: tensor toolbox

Journal Name	*Articles*
SIAM J APPL DYN SYST	32
SIAM J APPL MATH	548
SIAM J COMPUT	540
SIAM J CONTROL OPTIM	577
SIAM J DISCRETE MATH	260
SIAM J MATH ANAL	420
SIAM J MATRIX ANAL A	423
SIAM J NUMER ANAL	611
SIAM J OPTIMIZ	344
SIAM J SCI COMPUT	656
SIAM PROC S	469
SIAM REV	142

	Total in Collection	*Per Document*		
		Average	*Maximum*	*Minimum*
Unique terms	16617	148.32	831	17
abstracts	15752	128.06	802	11
titles	5164	10.16	33	1
keywords	5248	10.10	40	2
Authors	6891	2.19	13	1
Citations (within collection)	2659	0.53	12	0

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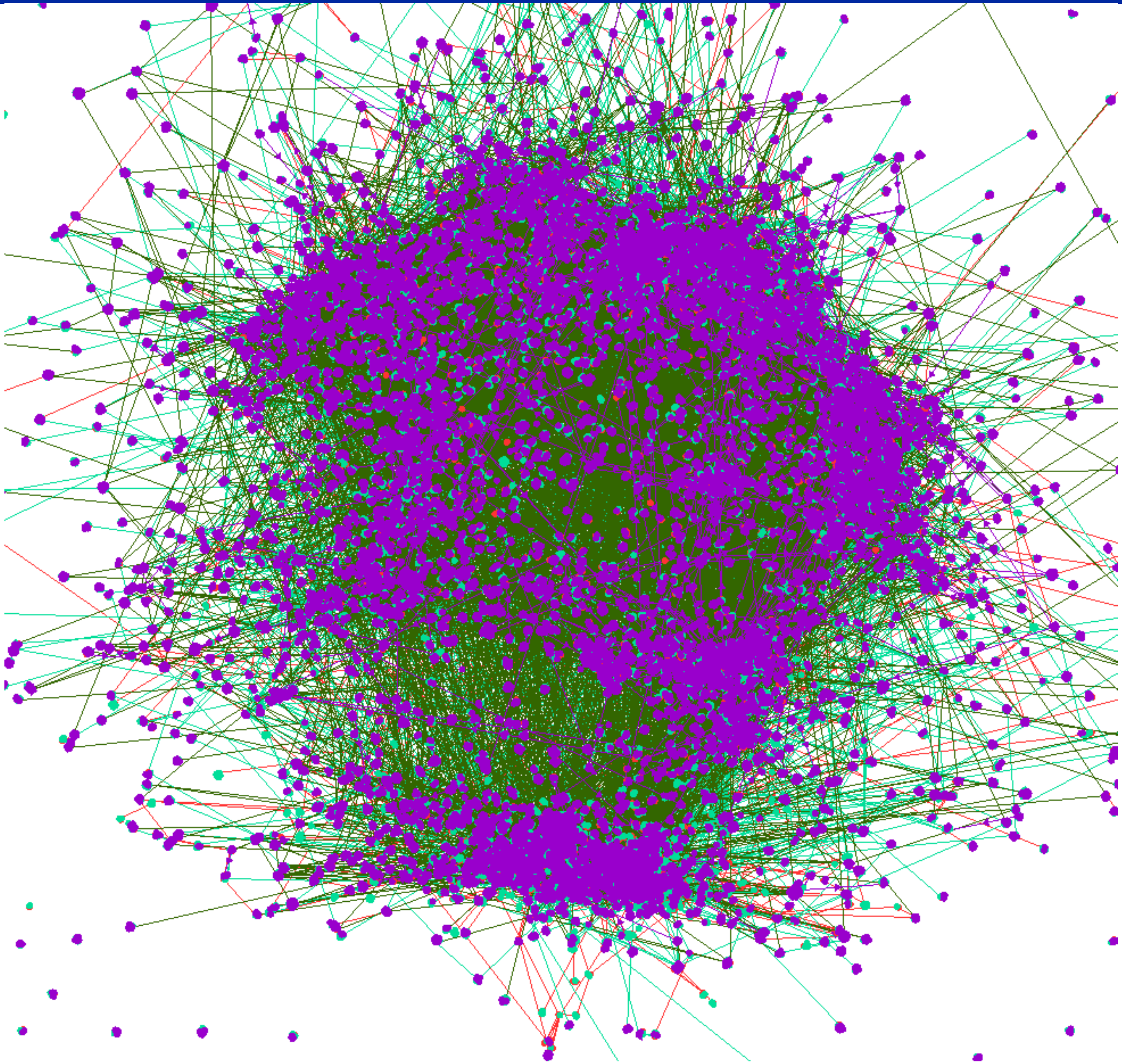
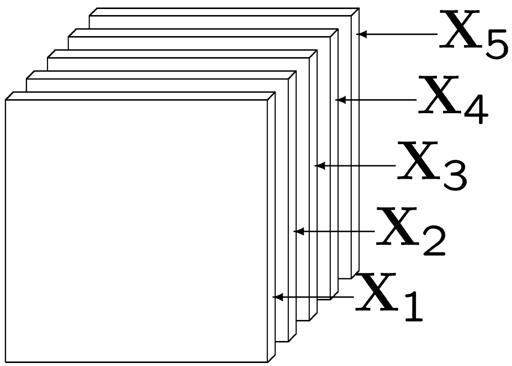
SIAM Data: Why Tensors?



SIAM Data: Tensor Construction



Frontal Slices $\mathbf{X}_k$

Slice (k)	*Description*	*Nonzeros*	$\sum_i \sum_j x_{ijk}$
1	Abstract Similarity	28476	7695.28
2	Title Similarity	120236	33285.79
3	Keyword Similarity	115412	16201.85
4	Author Similarity	16460	8027.46
5	Citation	2659	5318.00

- $\mathbf{X}_1 = \mathbf{T}^{\mathsf{T}}\mathbf{T}$ where $t_{ij} = f_{ij} \log_2(N/N_i)$ for terms in the **abstracts**
  - f_{ij} is the frequency of term i in document j
  - N_i is the number of documents that term i appears in
- $\mathbf{X}_2 = \mathbf{T}^{\mathsf{T}}\mathbf{T}$ for terms in the **titles**
- $\mathbf{X}_3 = \mathbf{T}^{\mathsf{T}}\mathbf{T}$ for terms in the author-supppplied **keywords**
- $\mathbf{X}_4 = \mathbf{W}^{\mathsf{T}}\mathbf{W}$ where $w_{ij} = \begin{cases} 1/\sqrt{M_j} & \text{if author } i \text{ wrote document } j \\ 0 & \text{otherwise,} \end{cases}$
- $\boxed{x_{ij5} = \begin{cases} 2 & \text{if document } i \text{ cites document } j \\ 0 & \text{otherwise.} \end{cases}}$

not symmetric

SIAM Data: CP Applications

$$\mathcal{X} \approx [\![\boldsymbol{\lambda} \,; \mathbf{A}, \mathbf{B}, \mathbf{C}]\!] = \sum_r \lambda_r \, \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$$

- **Community Identification**
  - Communities of papers connected by different link types

$$(\mathbf{a}_r, \ \mathbf{b}_r, \ \mathbf{c}_r)$$

- **Latent Document Similarity**
  - Using tensor decompositions for LSA-like analysis

$$\alpha \mathbf{A} \boldsymbol{\Lambda} \mathbf{A}^{\mathsf{T}} + \beta \mathbf{B} \boldsymbol{\Lambda} \mathbf{B}^{\mathsf{T}}$$

- **Analysis of a Body of Work via Centroids**
  - Body of work defined by query terms
  - Body of work defined by authorship

$$\alpha \mathbf{A} \boldsymbol{\Lambda} \mathbf{g}_A + \beta \mathbf{B} \boldsymbol{\Lambda} \mathbf{g}_B$$

- **Author Disambiguation**
  - List of most prolific authors in collection changes
  - Multiple author disambiguation

$$\alpha \mathbf{a}_i^{\mathsf{T}} \mathbf{g}_A + \beta \mathbf{b}_i^{\mathsf{T}} \mathbf{g}_B$$

- **Journal Prediction**
  - Co-reference information to define journal characteristics

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SIAM Data: Document Similarity

Link Analysis: Hubs and Authorities on the World Wide Web,
C.H.Q. Ding, H. Zha, X. He, P. Husbands, and H.D. Simon, *SIREV*, 2004.

- **CP decomposition, R = 10:** $\mathcal{X} \approx [\![\boldsymbol{\lambda} \, ; \mathbf{A}, \mathbf{B}, \mathbf{C}]\!] = \sum_{r=1}^{10} \lambda_r \, \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$
- **Similarity scores:** $\mathbf{S} = \frac{1}{2}\mathbf{A}\mathbf{A}^{\mathsf{T}} + \frac{1}{2}\mathbf{B}\mathbf{B}^{\mathsf{T}}$

Score	*Title*
0.000079	Ordering anisotropy and factored sparse approximate inverses
0.000079	Robust approximate inverse preconditioning for the conjugate gradient method
0.000077	An interior point algorithm for large-scale nonlinear programming
0.000073	Primal-dual interior-point methods for semidefinite programming in finite precision
0.000068	Some new search directions for primal-dual interior point methods in semidefinite programming
0.000068	A fast and high-quality multilevel scheme for partitioning irregular graphs
0.000067	Reoptimization with the primal-dual interior point method
0.000065	Superlinear convergence of primal-dual interior point algorithms for nonlinear programming
0.000064	A robust primal-dual interior-point algorithm for nonlinear programs
0.000063	Orderings for factorized sparse approximate inverse preconditioners

Interior Point Methods (Not related)
Sparse approximate inverses (Arguably distantly related)
Graph Partitioning (Related)

SIAM Data: Document Similarity

Link Analysis: Hubs and Authorities on the World Wide Web,

C.H.Q. Ding, H. Zha, X. He, P. Husbands, and H.D. Simon, *SIREV*, 2004.

- **CP decomposition, R = 30:** $\mathcal{X} \approx [\![\boldsymbol{\lambda}\,; \mathbf{A}, \mathbf{B}, \mathbf{C}]\!] = \sum_{r=1}^{30} \lambda_r \, \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$
- **Similarity scores:** $\mathbf{S} = \frac{1}{2}\mathbf{A}\mathbf{A}^{\mathsf{T}} + \frac{1}{2}\mathbf{B}\mathbf{B}^{\mathsf{T}}$

Score	*Title*
0.000563	Skip graphs
0.000356	Random lifts of graphs
0.000354	A fast and high-quality multilevel scheme for partitioning irregular graphs
0.000322	The minimum all-ones problem for trees
0.000306	Rankings of directed graphs
0.000295	Squarish k-d trees
0.000284	Finding the k-shortest paths
0.000276	On floor-plan of plane graphs
0.000275	1-Hyperbolic graphs
0.000269	Median graphs and triangle-free graphs

Graphs (Related)

SIAM Data: Author Disambiguation

- **CP decomposition, R = 20:** $\mathcal{X} \approx [\![\boldsymbol{\lambda}\,;\mathbf{A},\mathbf{B},\mathbf{C}]\!] = \sum_{r=1}^{20} \lambda_r \, \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r$
- **Computed centroids of top 20 authors' papers ($\mathbf{g}_A, \mathbf{g}_B$)**
- **Disambiguation scores:** $\mathbf{s} = \frac{1}{2}\mathbf{A}\mathbf{g}_A + \frac{1}{2}\mathbf{B}\mathbf{g}_B$
- **Scored all authors with same last name, first initial**

Before Disambuguation		*After Disambuguation*	
Papers	*Author*	*Papers*	*Author*
17	Du Q	17	Du Q
15	Kunisch K	16	Chan TF
15	Zwick U	16	Manteuffel TA
14	Chan TF	16	McCormick SF
13	Klar A	15	Kunisch K
13	Manteuffel TA	15	Zwick U
13	McCormick SF	13	Klar A
13	Motwani R	13	Golub GH
12	Golub GH	13	Motwani R
12	Kao MY	12	Kao MY

T Chan (2), TM Chan (4)

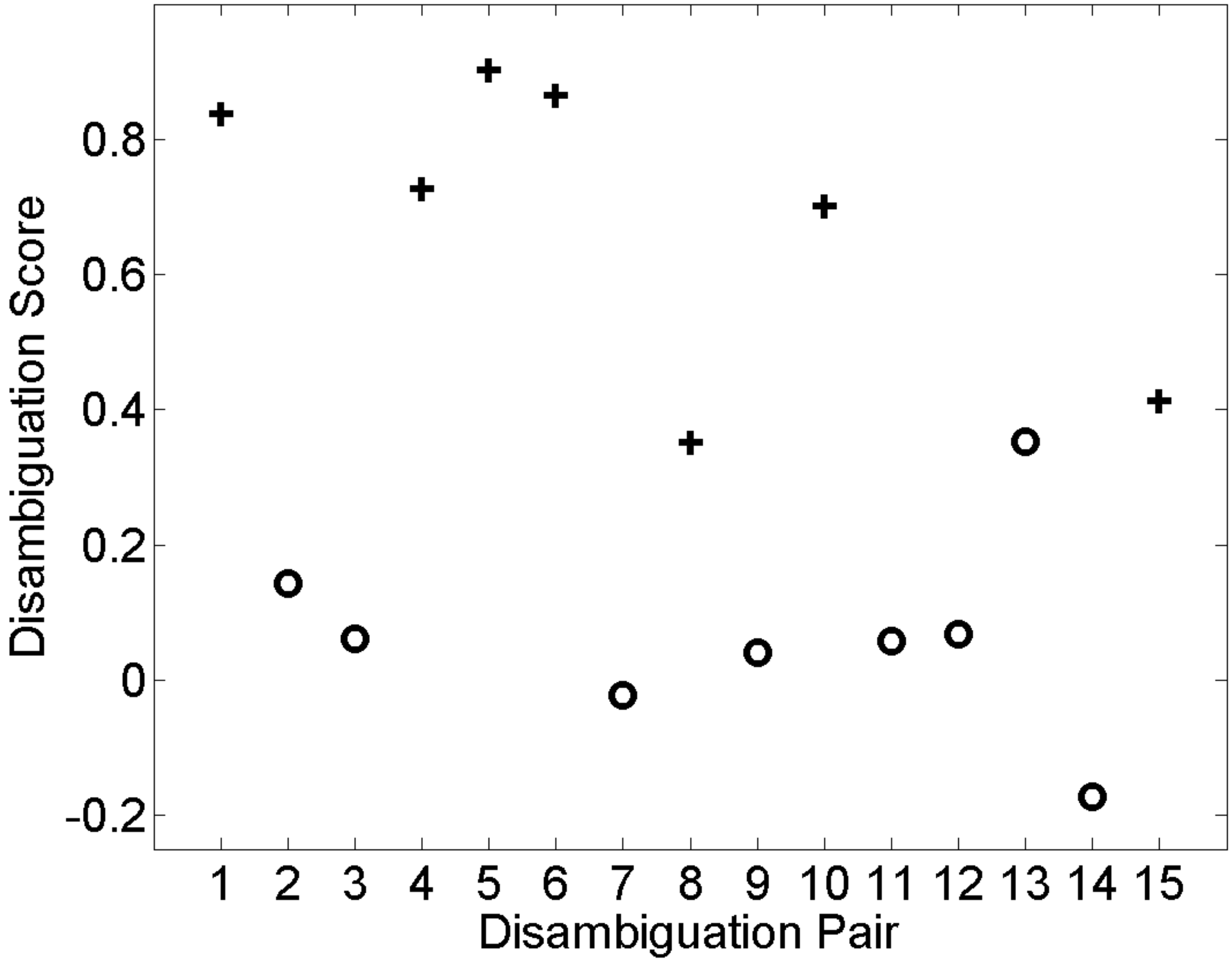
T Manteufel (3)

S McCormick (3)

G Golub (1)

SIAM Data: Author Disambiguation

Disambiguation Scores for Various CP Tensor Decompositions (+ = correct; o = incorrect)



***R* = 15**

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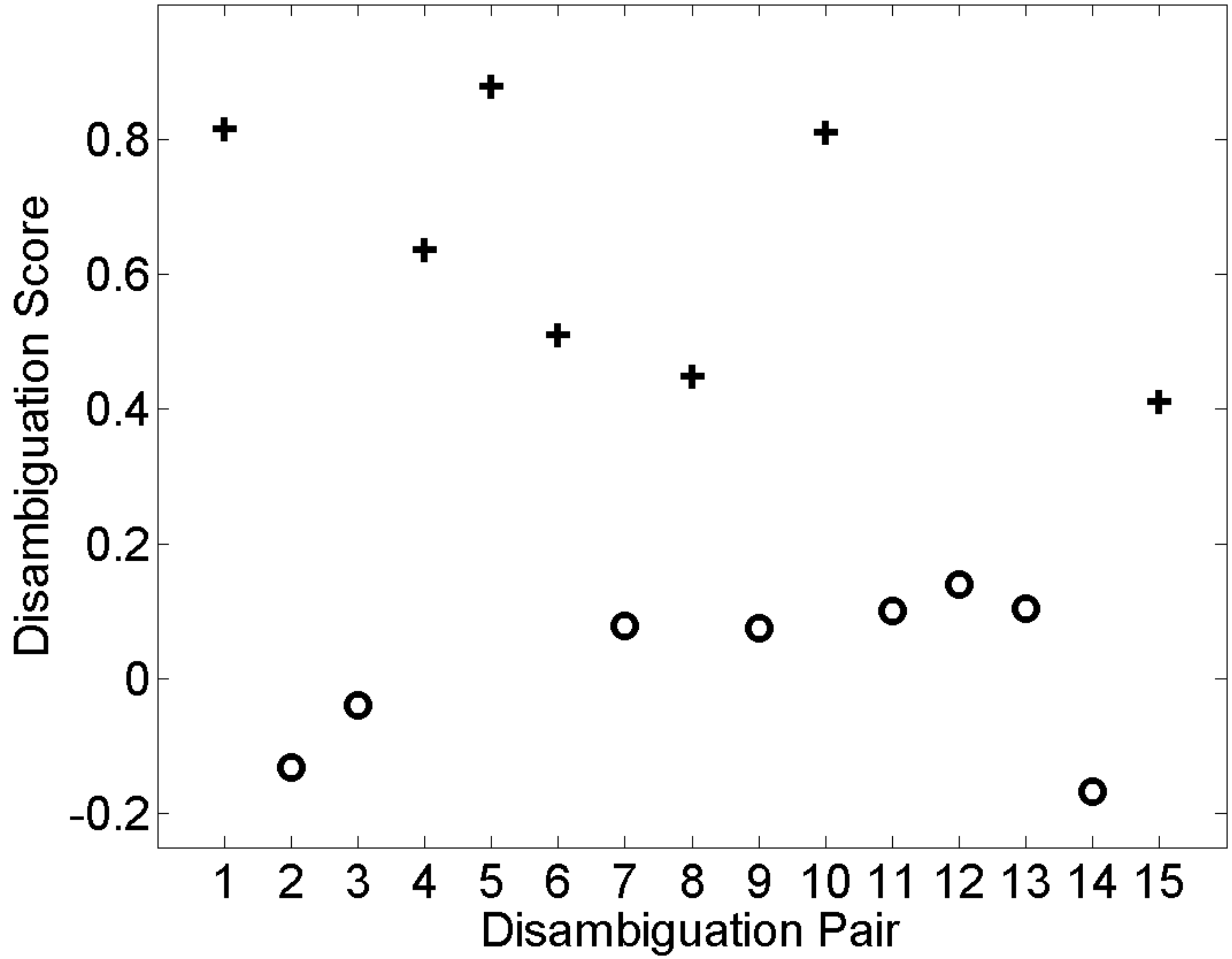
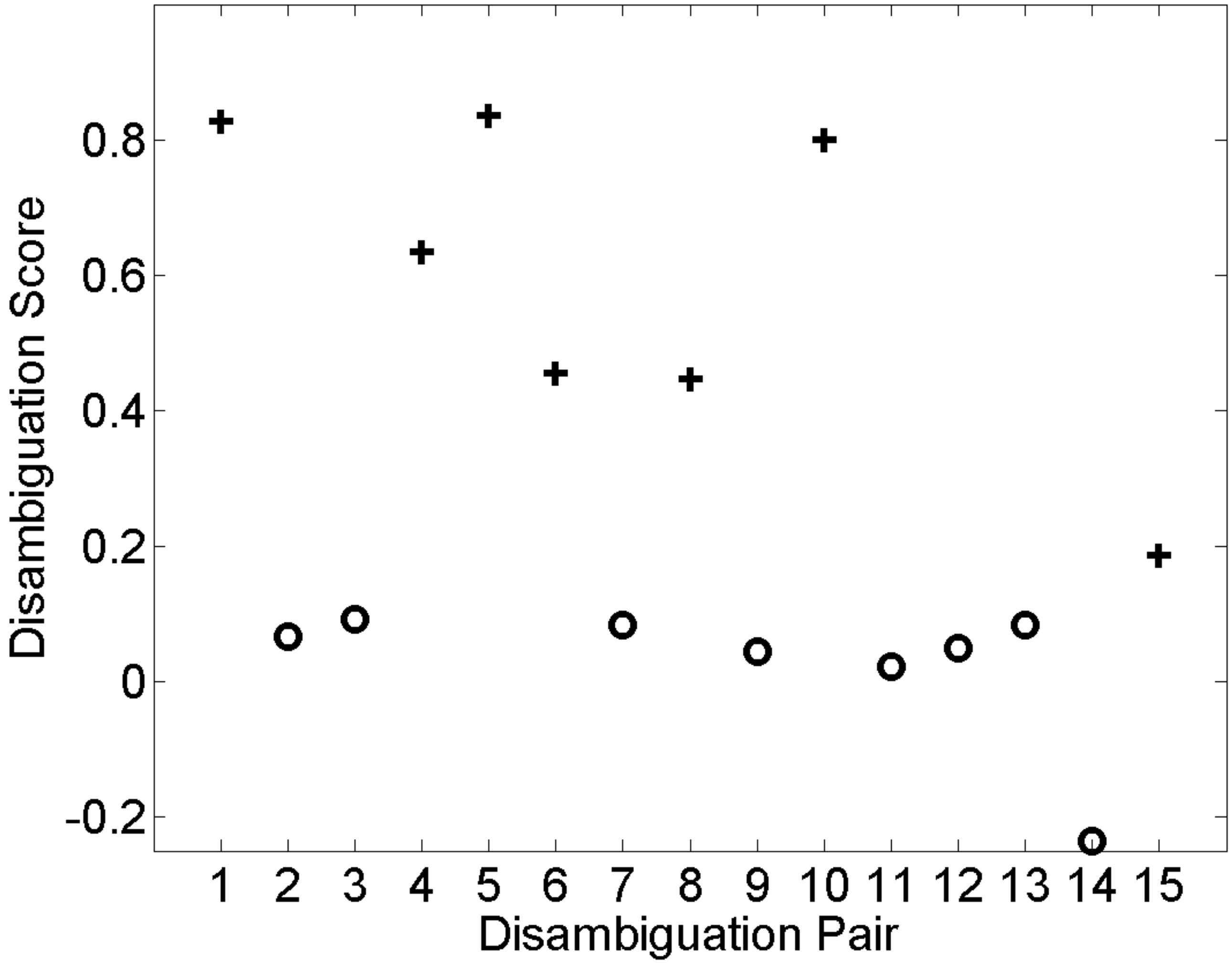
Disambiguation Scores for Various CP Tensor Decompositions (+ = correct; o = incorrect)



***R* = 20**

SIAM Data: Author Disambiguation

Disambiguation Scores for Various CP Tensor Decompositions (+ = correct; o = incorrect)



***R* = 25**

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SIAM Data: Journal Prediction

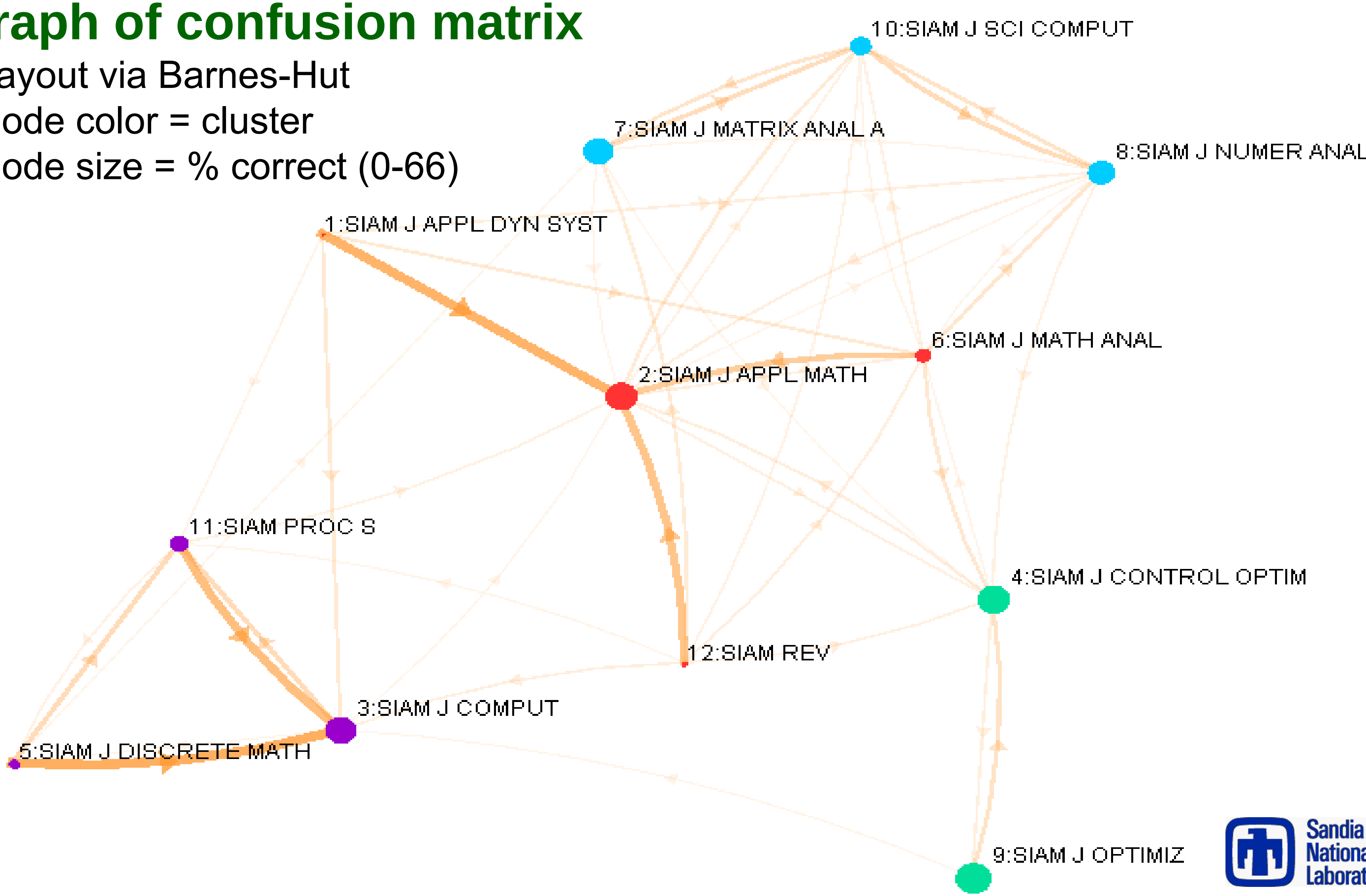
- **CP decomposition, R = 30, vectors from B**
- **Bagged ensemble of decision trees (n = 100)**
- **10-fold cross validation**
- **43% overall accuracy**

ID	*Journal Name*	*Size*	*Correct*	*Mislabeled as*
1	SIAM J APPL DYN SYST	1%	0%	2 (44%)
2	SIAM J APPL MATH	11%	58%	6 (10%)
3	SIAM J COMPUT	11%	56%	11 (20%)
4	SIAM J CONTROL OPTIM	11%	60%	2 (10%)
5	SIAM J DISCRETE MATH	5%	15%	3 (47%)
6	SIAM J MATH ANAL	8%	26%	2 (29%)
7	SIAM J MATRIX ANAL A	8%	56%	10 (19%)
8	SIAM J NUMER ANAL	12%	50%	10 (16%)
9	SIAM J OPTIMIZ	7%	66%	4 (16%)
10	SIAM J SCI COMPUT	13%	36%	8 (21%)
11	SIAM PROC S	9%	32%	3 (38%)
12	SIAM REV	3%	5%	2 (34%)

SIAM Data: Journal Prediction

Graph of confusion matrix

- Layout via Barnes-Hut
- Node color = cluster
- Node size = % correct (0-66)



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Tensors and HPC

- **Sparse tensors**
  - Data partitioning and load balancing issues
  - Partially sorted coordinate format
    - Optimize unfolding for single mode
- **Scalable decomposition algorithms**
  - CP, Tucker, PARAFAC2, DEDICOM, INDSCAL, …
    - Different operations, data partitioning issues
- **Applications**
  - Network analysis, web analysis, multi-way clustering, data fusion, cross-language IR, feature vector generation
    - Different operations, data partitioning issues
- **Leverage Sandia's Trilinos packages**
  - Data structures, load balancing, SVD, Eigen.

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Papers Most Similar to Authors' Centroids

John R. Gilbert (3 articles in data)

R		
20	30	Title
2	1	MSP A class of parallel multistep successive sparse approximate inverse preconditioning strategies
4	2	A factored approximate inverse preconditioner with pivoting
5	3	Algebraic multilevel methods and sparse approximate inverses
1	4	The recursive inverse eigenvalue problem
3	5	Efficient and stable solution of M-matrix linear systems of (block) Hessenberg form
19	6	A robust and efficient ILU that incorporates the growth of the inverse triangular factors
7	7	Orderings for factorized sparse approximate inverse preconditioners
8	8	Solving complex-valued linear systems via equivalent real formulations
18	9	On the relations between ILUs and factored approximate inverses
10	10	Scalable parallel preconditioning with the sparse approximate inverse of triangular matrices

Bruce A. Hendrickson (5 articles in data)

R		
20	30	Title
1	1	Norms of large Toeplitz band matrices
2	2	Special ultrametric matrices and graphs
4	3	A chart of backward errors for singly and doubly structured eigenvalue problems
3	4	Fast and stable algorithms for banded plus semiseparable systems of linear equations
6	5	Convex noncommutative polynomials have degree two or less
7	6	A bound for the inverse of a lower triangular Toeplitz matrix
9	7	Computing the minimum eigenvalue of symmetric positive definite Toeplitz matrix by Newton-...
10	8	Spectral characterizations for Hermitian centrosymmetric K-matrices and Hermitian skew-centro...
12	9	A Korovkin-based approximation of multilevel Toeplitz matrices (with rectangular unstructured ...
5	10	Recognizing perfect 2-split graphs

Thank You

Tensor Decompositions for Analyzing Multi-link Graphs

Danny Dunlavy

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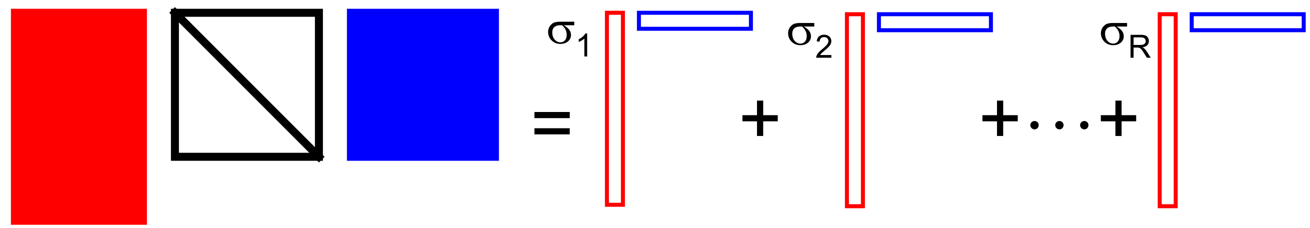
Extra Slides

High-Order Analogue of the Matrix SVD

- **Matrix SVD:**

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathsf{T}} =$$



- **Tucker Tensor (finding bases for each subspace):**

$$\mathbf{X} = \mathbf{\Sigma} \times_1 \mathbf{U} \times_2 \mathbf{V} = [\![\mathbf{\Sigma} \, ; \mathbf{U}, \mathbf{V}]\!]$$

- **Kruskal Tensor (sum of rank-1 components):**

$$\mathbf{X} = \sum_{r=1}^{R} \sigma_r \, \mathbf{u}_r \circ \mathbf{v}_r = [\![\boldsymbol{\sigma} \, ; \mathbf{U}, \mathbf{V}]\!]$$

Other Tensor Applications at Sandia

- **Tensor Toolbox (MATLAB)** [Kolda/Bader, 2007]
  http://csmr.ca.sandia.gov/~tgkolda/TensorToolbox/
- **TOPHITS (Topical HITS)** [Kolda, 2006]
  - HITS plus **terms** in dimension 3
  - Decomposition: CP
- **Cross Language Information Retrieval** [Chew et al., 2007]
  - Different **languages** in dimension 3
  - Decomposition: PARAFAC2
- **Temporal Analysis of E-mail Traffic** [Bader et al., 2007]
  - Directed e-mail graph with **time** in dimension 3
  - Decomposition: DEDICOM

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